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Effectiveness of resolvable incomplete blocks and extent of spatial relationships in variety testing trials on wheat

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SUMMARY

In the paper the effectiveness of resolvable incomplete block designs in variety testing trials on wheat is reported. The results of 122 trials conducted for variety testing purposes are used. The relative (average) variance of treatment comparisons is used as a measure of the effectiveness of incomplete blocks. The same trial data are also used in investigations of spatial relationships between measurements. Ten different models for semivariance were fitted and compared. Parameters of the best fitted model (in every single trial) were used to describe the features of spatial relationship between trial data.

Key words: effectiveness, geo-statistics, incomplete blocks, semivariance

1. Introduction

Use of incomplete block designs instead of the simpler-to-apply complete ones is often justified by the claim that they are more effective. This claim is in general true. Nevertheless, the superiority of incomplete blocks over complete ones depends on the type of incomplete blocks used (for example on the block size) and also on the variability of the experimental field used for experimentation. Often as a measure of the effectiveness of incomplete blocks the ratio of the average variance of treatment comparisons obtained for complete block design to the average variance obtained for incomplete blocks is used, see Patterson and Hunter (1983), Pilarczyk (1990). The most recent report on the effectiveness of resolvable incomplete blocks in Polish variety testing trials, based on numerous data, was given by Pilarczyk (1990). He showed that incomplete blocks were better than complete ones, but that their advantage over them was smaller than that observed in British trials reported by Patterson and Hunter (1983). This phenomena was explained by the fact that plot sizes (in particular plot width) were higher in the UK than in Poland. It can be expected that the effectiveness can also be affected by the set of varieties compared in the same trial. Because of recurrent replacement of older varieties by new ones, the set of varieties is now completely different than that used in previous report. So it is interesting to check again the effectiveness of incomplete blocks using modern data. On the other hand in many papers (see for example Papadakis (1937), Kempton and Howes (1981), Grondona and Cressie (1991) and many others) the basic assumption of the analysis of variance concerning independence of observations is questioned. To find the spatial dependence in trial data, methods that originated in mining (Journel and Huijbregts (1978)), known as "geo-statistical methods" are very often used (Kristensen and Ersbøll (1992), Martin (1986), McBratney and Webster (1986) and others). So the same trial data used for assessing the effectiveness of incomplete blocks were also used for detecting the most common shape of spatial relationship. Ten different models for semivariance were fitted and compared.

2. Data

The results of 122 trials performed in the years 1998 and 1999 on winter and spring wheat were used in this research. All but one trial were performed in resolvable incomplete block designs in r=4 replicates (superblocks). One trial was performed in a randomized complete block design. Plot size was always equal to 15 m^2 (1.5 m by 10 m). Superblocks were usually placed in four rows. The number of varieties varied from 27 to 48 depending on the type of wheat (spring or winter) and on the year. Block sizes varied between 5 and 8 plots depending on the number of varieties. In every single trial blocks of the same size, say k plots, or two sizes, k and (k-1) plots, were used. The mean harmonic efficiency factors (Pearce(1983)) of the applied designs were $\varepsilon=0.790$ in the worst cases and $\varepsilon=0.886$ in the best cases. The characteristic involved was the grain yield expressed in kg/plot and recalculated to common a moisture content of 15% .

3. Method

The results of every single trial were analyzed three times. The first time the subdivision of complete superblocks into incomplete blocks was disregarded. So every trial was treated as a randomized complete block trial. During the analysis the average variance of treatment comparisons was calculated and stored. Let σ^2_{ii} denote the average variance obtained in the i-th trial and let MS^v_{ei} denote the mean square error. Also residuals were calculated and stored for additional analysis using geo-statistical methods. In the second version the applied design was taken into account fully, but both variety and block effects were treated as fixed. This is equivalent to the use of the fixed effect model and application of so-called intra-block analysis of variance, Caliński and Kageyama (2002). Again the average variance of treatment comparisons was calculated and denoted by σ^2_{2i} and the relevant mean square error by MS^k_{ei}. In the third version of the analysis a mixed model of observation was applied (variety effects treated as fixed, block effects as random). This version of analysis is known in the literature (see Cochran and Cox(1956)) as the version with recovery of inter-block information. This time the average variance of treatment comparisons is denoted by σ^2_{3i}. The intra-block effectiveness of resolvable incomplete block design in i-th trial is calculated as

$$E_i = \sigma^2_{1i} / \sigma^2_{2i} \tag{1}$$

or equivalently, see Finney (1960), also Pilarczyk (1990) as

$$E_i = \mathrm{MS}^v_{ei} / (\mathrm{MS}^k_{ei} / \varepsilon_i), \tag{2}$$

where ε_i means the mean harmonic efficiency factor of the design applied in that trial. For the case when inter-block information is recovered, the effectiveness of incomplete blocks can again be calculated using formula (1) but with replacement of σ^2_{2i} by σ^2_{3i} or by the formula given by Patterson and Hunter (1983), which, using our symbols, has the form

$$\tilde{E}_i = \gamma\left(\varepsilon_i + \frac{(1-\varepsilon_i)(s-1)}{\gamma(v\text{-}1)-(v-s)}\right), \tag{3}$$

where $\gamma = \mathrm{MS}^v_{ei} / \mathrm{MS}^k_{ei}$, ε_i as in formula (2) is the mean efficiency factor of the design applied in i-th trial, v denotes the number of varieties, and s is the number of incomplete blocks within one superblock of this design.

To detect the existence and extent of spatial relationships, geo-statistical methods were applied. This consisted of calculation of semivariance for plots separated by h other plots with use classical formula (Webster (1985), Cressie (1993))

$$\hat{\gamma}(h) = \frac{1}{2m(h)} \sum_{j=1}^{r} \sum_{i=1}^{v-h} \left(e_{ij} - e_{i+h,j}\right)^2, \tag{4}$$

where m(h) denotes the number of cases where the difference h occurs, and e_{ij} denotes the residual obtained for i-th plots within the j-th superblock from the randomized complete block analysis of variance model. As indicated by formula (4), the semivariances were calculated for every distance within particular superblocks and then averaged over superblocks. The maximum distance for which semivariance was calculated was such that m(h) was at least 60. All the calculations at this stage were performed using the procedure FVARIOGRAM of the GENSTAT package (Genstat 5 Reference Manual (1989)). Next, using the obtained empirical semivariances, ten different models were fitted using weighted least squares method and realized by the procedure MVARIOGRAM from the same package. In our approach, spatial dependence within a trial was treated as detected if the applied iterative procedure converged and the obtained parameters were feasible (for example, range higher than 1 and partial sill and sill positive). From among all fitted models for every particular trial the best fitted model was then chosen (using the coefficient of determination criterion). Every trial was characterized by that the best fitted model.

4. The results

4.1 Effectiveness

The results of the preliminary calculations are presented in Table 1. Apart from mean values of E_i and $\tilde{E}_i$, adjusted mean values distinguished by an asterisk are also given. These adjusted mean were calculated in such a way that all effectiveness values smaller than one were replaced by 1.000. This means that in all cases where incomplete blocks were less efficient than a randomized complete block design, the trial was treated as one performed in complete blocks. This is justified by the common practice (at least in variety testing) that in all cases where incomplete blocks are inefficient the division of complete superblocks into incomplete blocks is disregarded. The incomplete blocks were approximately for 20% more effective than complete ones. This result is in accordance with the earlier results of Pilarczyk (1990), who reported the mean values of $\overline{E}$ and $\overline{E}^*$ as 1.134 and 1.314 respectively. For intra- and inter- block analysis of variance, the fraction of trials with higher efficiency of incomplete blocks than complete ones was 68% and 96% respectively. In seven trials out of 121 the efficiency of incomplete blocks was higher than 2.000.

Table 1. Effectiveness of incomplete blocks in intra- and inter-block analysis of variance

Species	V	No of trials	Block size	$\overline{E}$	$\overline{E}^*$	$\widetilde{E}$	$\widetilde{E}^*$
1998							
Winter wheat	48	7	8	1.036	1.068	1.103	1.103
	38	16	8	1.049	1.107	1.131	1.131
	32	18	8	1.199	1.246	1.266	1.266
Spring wheat	28	22	7 and 5	1.238	1.269	1.307	1.307
	36	6	6	1,396	1,447	1,518	1,518
1999							
Winter wheat	42	16	6 and 7	1.100	1.143	1.186	1.186
	39	9	8	1.067	1.104	1.133	1.134
	27	6	7	1.214	1.241	1.277	1.277
Spring wheat	29	21	8	1.196	1.237	1.274	1.274
Total		121		1.163	1.204	1.239	1.239

4.2 Variation versus effectiveness of incomplete blocks

To determine the dependence of effectiveness of incomplete blocks on the variability of experimental fields, additional analysis was made. The trials were divided into two groups according to their coefficients of variation. The first group (denoted by A) was formed from all trials with smaller than median coefficients of variation. The second group (denoted by B) consists of all trials with higher than median coefficients of variation. For both these groups the same parameters as for the whole set of trials (given in Table 1) were calculated. These new values are presented in Table 2.

Table 2. Effectiveness of incomplete blocks related to field variability expressed by coefficients of variation

	$\overline{E}$	$\overline{E}^*$	$\widetilde{E}$	$\widetilde{E}^*$	Number of trials with	
					$\overline{E}<1$	$\overline{E}>2$
group A	1,029	1,086	1,121	1,121	30	0
group B	1,295	1,321	1,355	1,355	16	7

Among trials with smaller coefficients of variation (group A) there are as many as 30 trials with inefficient incomplete blocks and there is no trial with effectiveness higher than 2.000. On the other hand, among trials with higher coefficients of variation (group B) there are only 16 trials for which incomplete blocks were inefficient and seven trials with effectiveness of incomplete blocks higher than 2.000. It is worth mentioning that the effectiveness of the analyses can be additionally improved by application of one of the spatial models (see Pilarczyk and Tomaszyk (2006)). For example the average (over all trials) effectiveness of a randomized complete block model supplemented by a linear structure within a variance-covariance matrix was 1.32. The linear structure was introduced because the linear and bounded linear models were those most often successfully fitted when different models were fitted in turn to our trial data.

The other interesting feature of the results obtained is that the effectiveness of incomplete blocks is more related to the coefficient of variation (variability of experimental fields) than to the number of varieties compared.

4.3 Spatial relationships

All 122 trials were analysed in turn using the procedure FVARIOGRAM for calculating empirical semivariances up to a distance V-15 plots. This constraint was imposed to ensure that the minimum number of observations for each distance was at least 60. In the next step ten different models, proposed in the literature (Cressie (1993), Journell and Huijbregts (1978)), were fitted to the empirical semivariances. The following models were fitted: spherical, bounded linear, circular, pentaspherical, double-spherical, exponential, Gauss, Bessel, power and linear. Parameters of these models were calculated using the weighted least squares methods. From among all fitted models the best one was always chosen using the coefficient of determination as the criterion (it is worth mentioning that exactly the same results were obtained when instead of coefficient of variation the Akaike information criterion was applied (Cressie (1985))). The parameters of the best fitted model were next used to describe the extent of spatial relationship (coefficient of determination) and the range of relationship. The results are shown in Table 3.

Spatial relationships among data were detected in 80% of all trials (in 98 out of 122 trials). How it depends on the number of varieties, type of wheat and year of trailing is shown in the first four columns of Table 3. As we can see, no meaningful differences were observed either between types of wheat or between years. In order to characterize the part of variability explained by spatial models, all trials were split into three groups. First one consist of trials with relatively poor (coefficient of determination $R^2 \leq 30\%$) fitting of the best spatial model. The second group consist of trials with medium fitting ($30\% < R^2 \leq 70\%$). Finally the third group contains trials with good fitting ($R^2 > 70\%$). Again, independently of the year of trailing, the sizes of second and third group were approximately the same. The first group was the smallest. In the last three columns of Table 3 the

Table 3. Number of trials with detected (and fitted) spatial dependence and some of its parameters

Species	Number of varieties	Number of trials		Number of trials with (for the best fitted model)					
		All	With spatial dependence detected by at least one model	Coefficient of determination R^2 in percentages			Range a		
				$R^2 \leq 30$	$30 < R^2 \leq 70$	$R^2 > 70$	$a \leq 7$	$7 < a \leq 14$	$a > 14$
1998									
Winter wheat	48	7	6	4	2	0	1	2	3
	38	16	11	3	4	4	1	2	8
	32	18	12	1	5	6	1	2	9
	Total	41	29	8	11	10	3	6	20
Spring wheat	28	22	20	2	9	9	4	14	2
	36	6	5	0	2	3	0	1	4
	Total	28	25	2	11	12	4	15	6
1999									
Winter wheat	42	16	13	1	9	3	4	2	7
	39	10	10	5	2	3	3	1	6
	27	6	5	1	0	4	0	5	0
	Total	32	28	7	11	10	7	8	13
Spring wheat	29	21	16	1	6	9	2	13	1
	Total	21	16	1	6	9	2	13	1
Total		**122**	**98**	**18**	**39**	**41**	**16**	**42**	**40**

numbers of trials with different values of range (maximum distance with spatial dependence) are given. Again all trials were split into three groups. The first group includes all trials with range of spatial dependence $a \leq 7$, the second group consist of trials with the range a between 7 and 14 ($7 < a \leq 14$), and the last group contains trials with the range $a > 14$. The distance of 7 plots was chosen as a boundary value because in the majority of analysed trials the block size was 7 (or 8) plots. For unbounded models the range was taken to the maximum distance considered in calculation, whereas for asymptotically bounded models the range was such value of distance for which the semivariance reached the value of nugget plus 95% of partial sill (Webster (1985)). The values given in Table 3 show that if there are spatial relationships between data, the range is usually larger than the commonly used block size.

5. Comments and conclusions

The analysis of extensive trial data on spring and winter wheat confirmed earlier results concerning the effectiveness of incomplete blocks relative to randomized complete block designs. For the analysed set of data, resolvable incomplete blocks were found to reduce the variance of differences between varieties by 20% compared to randomized complete blocks. The advantage of incomplete blocks was higher in trials performed in more heterogeneous fields (in trials characterized by higher coefficients of variation). In a majority of trials (in about 80% of them) one or more spatial models were successfully fitted. Thus the fallacy of one of the assumption of analysis of variance concerning independence of observations was again confirmed. The range of spatial dependence in trial data was usually larger than that caused by the size of the incomplete blocks used.

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